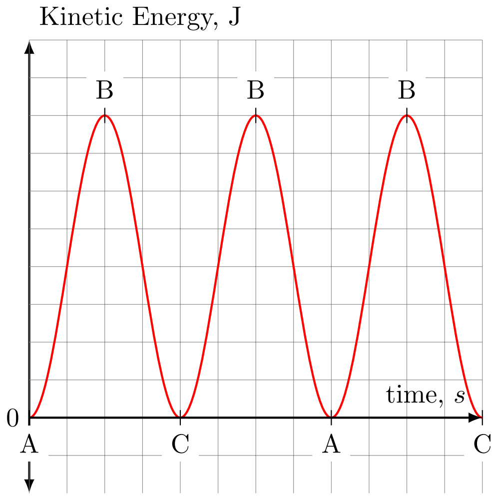
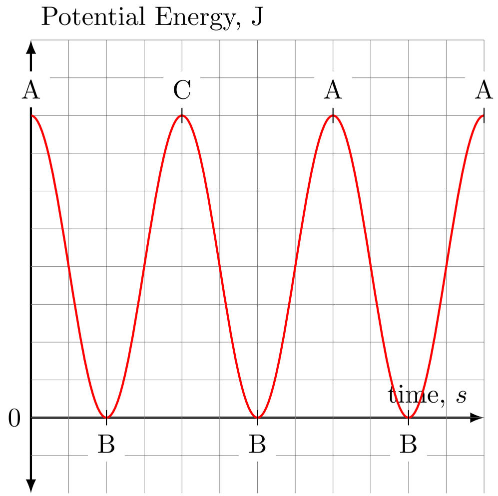
Name

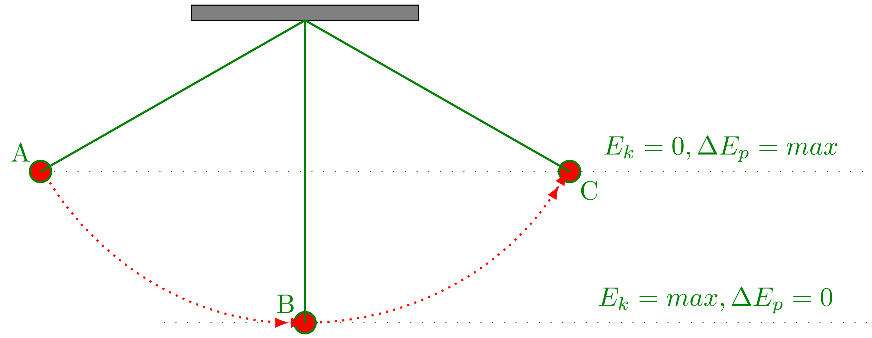
Date

THE VAULT
www.rawsonvault.com

SHM Kinematics 3 (4.2.2)







Deriving E_k

- From Kinematics 1, $\mathbf{v = -v_o sin\omega t}$
- As $E_k = \frac{1}{2}mv^2$, E_k is now equal to

$$E_k = \frac{1}{2}mv_o^2 sin^2\omega t$$

Deriving ΔE_p

- We can deduce that ΔE_p has an opposite magnitude as E_k, but follows a cosine^2 curve
- As $E_k = \frac{1}{2}mv_o^2 sin^2\omega t$

$$\Delta E_p = \frac{1}{2}mv_o^2 cos^2\omega t$$

- Graphically you can see that E_k and ΔE_p add together to create a constant value, the *Total Mechanical Energy*
- This also parallels Pythagoras:
  $1 = cos^2\theta + sin^2\theta$

Deriving $\mathbf{v}_{max} = \omega\mathbf{x}$

$$1 = cos^2\theta + sin^2\theta$$
$$sin\theta = \sqrt{1 - cos^2\theta}$$
$$sin\omega t = \sqrt{1 - cos^2\omega t}$$
$$\omega x_o sin\omega t = \omega x_o\sqrt{1 - cos^2\omega t}$$
$$\omega x_o sin\omega t = \omega\sqrt{x_o^2 - x_o^2 cos^2\omega t}$$
$$x^2 = x_o^2 cos^2\omega t \quad \text{so}$$
$$v = \omega\sqrt{x_o^2 - x^2}$$

The maximum velocity is when displacement x=0 so therefore

$$v_{max} = \omega x$$

Deriving $E_{k,max}, \Delta E_p$

$$E_k = \frac{1}{2}mv^2$$
$$v = \omega\sqrt{x_o^2 - x^2}$$
$$E_k = \frac{1}{2}m\omega^2(x_o^2 - x^2)$$

The maximum E_k is when displacement x=0 so therefore

$$E_{k,max} = \frac{1}{2}m\omega^2(x_o^2)$$
$$\therefore \text{totalenergy} = \frac{1}{2}m\omega^2(x_o^2)$$

$$\Delta E_p = \text{totalenergy} - E_k$$
$$\Delta E_p = \frac{1}{2}m\omega^2(x_o^2) - \frac{1}{2}m\omega^2(x_o^2 - x^2)$$

$$\Delta E_p = \frac{1}{2}m\omega^2(x^2)$$